Name ______________________________ Q1 Test 3 Review Sheet

1. Write the equations of the tangent line to the curve $f(x) = -\sin(x)$ when $x = \pi$.

2. Given $h(x) = (3x^3 - x^2 + 10x + 2)\cos(x)$, find $h'(x)$.

3. Find the derivative of the following function in simplest form: $y = \frac{3x^2 - 2}{2x - 3}$

For questions 4 – 7 use the following table to find y' at x = 1, if:

f(1)	f ' (1)	g(1)	g ' (1)
3	4	1	-2

4. $y = f(x)g(x)$

$y' = f(x)\cdot g'(x) + f'(x)\cdot g(x)$

$= (3)(-2) + (4)(1) = \boxed{-2}$

5. $y = \frac{f(x)}{g(x)}$

$y' = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2} = \frac{(1)(4)-(3)(-2)}{1^2} = \boxed{10}$

6. $y = x^4 g(x)$

7. $y = \frac{x^3 - 2x}{g(x)}$

8. Find the coordinates of the point(s) where $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$ has horizontal tangents.

9. Find the equation of the tangent line to the curve, $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$ when x = -2

10. Find $f'(x)$ if $f(x) = \frac{3x^2}{x-1}$

11. Find $f'(x)$ if $f(x) = 3x^2 \sin x$

Find the derivative of each of the following:

12. $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

13. $f(x) = \sin(3x+1)$

14. $f(x) = \sqrt[4]{(x^2+5x)^3}$

15. $y = \ln x^5$

16. $y = e^{4x^3+2}$

17. $y = x^5 - \ln(x) + 5e^2$

15) $y = \ln(x^5)$

$$\frac{dy}{dx} = \frac{1}{x^5} \cdot 5x^4 = \frac{5x^4}{x^5} = \boxed{\frac{5}{x}}$$

16) $y = e^{4x^3+2}$

$$\frac{dy}{dx} = e^{4x^3+2} \cdot 12x^2 = \boxed{12x^2 e^{4x^3+2}}$$

17) $y = x^5 - \ln(x) + 5e^2$

constant

$$\frac{dy}{dx} = 5x^4 - \frac{1}{x} + \cancel{5e^2 \cdot (0)}$$

$$\frac{dy}{dx} = \boxed{5x^4 - \frac{1}{x}}$$

6) $y = x^4 \cdot g(x)$

$\frac{dy}{dx} = x^4 \cdot g'(x) + 4x^3 \cdot g(x)$

$= x^4(-2) + 4x^3(1)$

$\frac{dy}{dx} = -2x^4 + 4x^3$

$*$ $x=1$

$= -2(1)^4 + 4(1)^3$

$= -2+4$

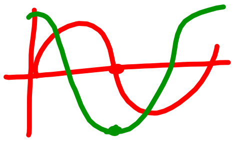
$= \boxed{2}$

7) $y = \frac{x^3-2x}{g(x)}$

$\frac{dy}{dx} = \frac{g(x)\cdot(3x^2-2) - (x^3-2x)\cdot g'(x)}{(g(x))^2}$

$= \frac{(1)(3(1)^2-2) - (1^2-2(1))\cdot(-2)}{1^2} = \frac{1-2}{1} = \boxed{-1}$

1) $f(x) = -\sin(x)$



$x = \pi$

Point: use the function

$f(\pi) = -\sin(\pi)$

$f(\pi) = 0$

$(\pi, 0)$

Slope: use the Derivative

$f'(x) = -\cos(x)$

$f'(\pi) = -\cos(\pi)$

$f'(\pi) = -(-1)$

$f'(\pi) = 1$

Slope $= 1$

$y - 0 = 1(x - \pi)$

$\boxed{y = x - \pi}$

8) $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

$f'(x) = x^3 - x^2 - 2x$

$0 = x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= x(x-2)(x+1)$

$x = 0 \quad x = 2 \quad x = -1$

$f(0) = \frac{1}{4}(0)^4 - \frac{1}{3}(0)^3 - (0)^2$

$= 0$

$(0, 0)$

$f(2) = \frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - 2^2$

$= 4 - \frac{8}{3} - 4$

$= -\frac{8}{3}$

$(2, -\frac{8}{3})$

$f(-1) = \frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - (-1)^2$

$= \frac{1}{4} + \frac{1}{3} - 1$

$= -\frac{5}{12}$

$(-1, -\frac{5}{12})$

Product rule!

2) $h(x) = (3x^3 - x^2 + 10x + 2)(\cos(x))$

$$\boxed{h'(x) = (3x^3 - x^2 + 10x + 2)(-\sin(x)) + (9x^2 - 2x + 10)(\cos(x))}$$

3) $y = \dfrac{3x^2 - 2}{2x - 3}$ ← Quotient

$\times$ $+$

$\div$ $-$

$$\frac{dy}{dx} = \frac{(2x-3)(6x) - (3x^2-2)(2)}{(2x-3)^2} =$$

$$= \frac{12x^2 - 18x - 6x^2 + 4}{4x^2 - 12x + 9} = \boxed{\frac{6x^2 - 18x + 4}{4x^2 - 12x + 9}}$$

8) Horizontal tangents when derivative $=0$

$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

$f'(x) = x^3 - x^2 - 2x$

$0 = x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= x(x-2)(x+1)$

$x=0 \quad x=2 \quad x=-1$

$f(0)=0$

$f(2) = -\frac{8}{3}$

$f(-1) = -\frac{5}{12}$

$$\boxed{\begin{array}{c}(0,0)\\ \left(2,-\frac{8}{3}\right)\\ \left(-1,-\frac{5}{12}\right)\end{array}}$$

9) $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

Point:

$f(-2) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

$f(-2) = \frac{8}{3}$

$\left(-2, \frac{8}{3}\right)$

Slope:

$f'(x) = x^3 - x^2 - 2x$

$f'(-2) = -8 - 4 + 4$

$f'(-2) = -8$

$$\boxed{y - \frac{8}{3} = -8(x+2)}$$

10) $f(x) = \frac{3x^2}{x-1}$

$3x^2(x-1)^{-1}$

$f'(x) = \frac{(x-1)(6x) - (3x^2)(1)}{(x-1)^2}$

$= \frac{6x^2 - 6x - 3x^2}{x^2 - 2x + 1} = \boxed{\frac{3x^2 - 6x}{x^2 - 2x + 1}}$

10) Alt: $f(x) = \frac{3x^2}{x-1}$

$$f(x) = \boxed{3x^2} \cdot \boxed{(x-1)^{-1}}$$

$$f'(x) = 3x^2 \cdot (-1)(x-1)^{-2} \cdot (1) + 6x(x-1)^{-1}$$

$$-3x^2 \cdot (x-1)^{-2} + 6x(x-1)^{-1}$$

$$\frac{-3x^2}{(x-1)^2} + \frac{6x}{(x-1)} \frac{(x-1)}{(x-1)}$$

$$\frac{-3x^2 + 6x^2 - 6x}{(x-1)^2}$$

$$\frac{3x^2 - 6x}{x^2 - 2x + 1}$$

11) $f(x) = 3x^2 \cdot \sin(x)$

$$\boxed{f'(x) = (3x^2)(\cos(x)) + (\sin(x))(6x)}$$

12) $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

$f(x) = 5x + 2x^{1/3} - 3x^{-2}$

$f'(x) = 5 + \frac{2}{3}x^{-\frac{2}{3}} + 6x^{-3}$

$= 5 + \frac{2}{3\sqrt[3]{x^2}} + \frac{6}{x^3}$

chain Rule

13) $f(x) = \sin(3x+1)$

$f'(x) = \cos(3x+1) \cdot (3)$

$= 3\cos(3x+1)$

14) $f(x) = \sqrt[4]{(x^2+5x)^3} =$

$f(x) = (x^2+5x)^{3/4}$

$f'(x) = \frac{3}{4}(x^2+5x)^{-1/4} \cdot (2x+5)$

$f'(x) = \frac{3(2x+5)}{4(x^2+5x)^{1/4}} = \frac{6x+15}{4\sqrt[4]{x^2+5x}}$

Extra:

23) Given $f(x) = \sqrt[7]{x^5+3x^4}$ find $f'(2)$

$$f(x) = (x^5+3x^4)^{1/7}$$

$$f'(x) = \frac{1}{7}(x^5+3x^4)^{-\frac{6}{7}} \cdot (5x^4+12x^3)$$

$$f'(x) = \frac{1(5x^4+12x^3)}{7(x^5+3x^4)^{6/7}} = \frac{(5x^4+12x^3)}{7\sqrt[7]{(x^5+3x^4)^6}}$$

$$f'(2) = \frac{1}{7}(2^5+3(2)^4)^{-\frac{6}{7}}(5(2)^4+12(2)^3)$$

$$f'(2) = .58775$$

24) Find $\frac{d}{dx}\left(\ln(4x^2) + e^{(2x^2-5x)}\right)$

$$\frac{d}{dx} = \frac{1}{4x^2} \cdot 8x + e^{2x^2-5x} \cdot (4x-5)$$

$$= \frac{8x}{4x^2}$$

$$= \frac{2}{\cancel{x}} + (4x-5)e^{2x^2-5x}$$

25) $y = \boxed{3x^2} \cdot \boxed{\sin(x^3)}$

$y' = 3x^2\left(\cos(x^3) \cdot 3x^2\right) + 6x \cdot \sin(x^3)$

$= 9x^4 \cdot \cos(x^3) + 6x\sin(x^3)$